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Active Loop Aerials for HF Reception Part 1: Practical Loop Aerial Design

Should you consider a small loop antenna? Why? Can they be efficient?

By Chris Trask, N7ZWY

The performance of any receiving system is highly dependent on the first few stages. Very often the aerial itself is not given consideration as being a part of that system, let alone recognized as the stage that determines the minimum noise figure (*NF*) that can be realized. Aerials having poor efficiency or that are not properly matched to the receiving unit can cripple the ability of the operator to receive low-level signals. Electrically small aerials, regardless of their configuration, are especially vulnerable to low efficiencies and hence become the critical elements in their receiving systems where larger aerials with good efficiency are impractical. With many radio amateurs and shortwave listeners (SWLs) living in apartments or subject to property zoning restrictions, small aerials having good performance, both for transmitting and receiving, have become increasingly popular. This two-part series will describe the theory and practical design of a high-performance active receiving loop aerial.

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Active Aerials

Active aerials are often considered as being a viable option when the need for a small aerial arises. Many such aerials are available commercially, and most that are published for hobbyists fail to provide the operator with good performance for many reasons. The two most relevant are that the antenna itself is of poor efficiency, and the aerial amplifier has poor dynamic range, either with respect to *NF* or intermodulation distortion (*IMD*). Very often, it is both maladies.

Short vertical monopoles and dipoles are notorious for poor efficiency, and it does not help to follow a poorly designed aerial with a high-dynamic-range amplifier, regardless of how low the amplifier's *NF* may be. The aerial itself—or rather the aerial noise temperature—determines the minimum *NF* of the receiving system. This is somewhat contrary to the notion that the use of small tuned loops for receiving can be justified when taking the expected external noise into account.[1] To those having only basic skills who want to realize a small aerial with good efficiency, very few options are

[1]Notes appear on page 41.

as easily comprehended and constructed as the loop aerial. When properly designed and constructed, a loop aerial can easily exceed 80% efficiency. The low real impedance of the loop, usually less than an ohm, is in fact a highly desirable feature as exceptionally high tuning Q's can be realized. Loop aerials also have good directivity, whereas the vertical monopole is unidirectional.

The low impedances of loop aerials can be easily dealt with by way of suitable aerial amplifiers that are designed with due regard to matching these low impedances. The remaining necessary features of low *NF* and good *IMD* performance can be attained with equal ease. Remote tuning is also a desirable feature to accommodate the reactance of the loop aerial and to give some degree of selectivity at the earliest stage of the receiving system. The active loop aerial described here consists of three basic parts: the loop aerial proper, the aerial amplifier unit, which includes remote tuning and the aerial control unit.

Loop Aerials

Again, it does not help in any way to follow a poorly designed aerial with a high-dynamic-range amplifier regardless of how low the amplifier's *NF* may be. Therefore, it is necessary to look much more closely at the loop aerial. We should become familiar with the various design aspects and their effects on performance so as to arrive at a suitable design that focuses on the efficiency of the aerial, very likely at the expense of broadband operation.

Generally, two loop-aerial models are popular. The first of these, shown in Fig 1, consists of a signal voltage source, an inductance L_{ant}, a radiation resistance R_{ant} and a loss resistance R_{loss}. This model is suitable for loop aerials that are very small with respect to a wavelength, the guideline is that the enclosed area of the aerial should be much less than the operating wavelength squared. For this model, the inductance is easily determined by way of classical inductance formulas,[2] and it has been suggested that the radiation resistance of a loop aerial of any geometry can be determined by:

$$R_{ant} = 320\,\pi^4 \left(\frac{A}{\lambda^2}\right)^2 N^2 \quad \text{(Eq 1)}$$

where A is the area of the aerial and λ is the wavelength, both in the same units,[3, 4] and N is the number of turns. This formula is convenient for frequencies that are much less than the first antiresonance frequency of the aerial, but it loses significance as the frequency is increased towards antiresonance and fails completely at antiresonance. Kraus notes (see Note 4) that Eq 1 is applicable only for very small loops where $A < (\lambda^2 / 100)$.

For small loop aerials, the loss resistance is primarily a result of skin effect and can be approximated by (see Note 4):

$$R_{loss} = \frac{L}{\sigma\,\pi\,d\,\delta} = \frac{L}{d}\sqrt{\frac{f\mu_0}{\pi\sigma}} \quad \text{(Eq 2)}$$

where f is the frequency in Hertz, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m), σ is the conductivity of the material in S/m, L is the perimeter length of the aerial and d is the diameter of the conductor, both in meters. The quantity δ is referred to as the *depth of penetration* (see Note 4), and is defined as:

$$\delta = \frac{1}{\sqrt{f\pi\mu\sigma}} \quad \text{(Eq 3)}$$

where μ is the permeability of the material in H/m. From Eq 2, it can be seen that the loss resistance can be improved by increasing the size of the conductor. It might also be improved by the use of Litz wire; but because of irregularities in the stranding and capacitance between the strands, the effectiveness of Litz wire in reducing the loss resistance diminishes above 1 MHz.[5]

Power applied to the aerial is dissipated as electromagnetic energy by the radiation resistance and as heat by the loss resistance. The ratio of radiated power to total power is referred to as the aerial efficiency and is readily defined as:

$$Eff = \frac{R_{ant}}{R_{ant} + R_{loss}} \quad \text{(Eq 4)}$$

When we measure the terminal impedance of the antenna, we see these two resistances as the real part of the impedance. These resistances can be determined by measuring the terminal impedance and the efficiency of the aerial, the latter of which can be measured two methods,[6] both of which require test equipment beyond the means of most amateurs and hobbyists. For the purposes discussed here, an analytical approach is more desirable.

A more-exact model of the loop aerial is shown in Fig 2, where the inductance of the model in Fig 1 has been replaced with a λ/4 transmission line of impedance Z_0. This model holds true for loop aerials with balanced feeds, and it helps to explain the various antiresonances and resonances that are observed in practice. For loop aerials with unbalanced feeds, the transmission line becomes λ/2, which has serious consequences with respect to impedances and radiation patterns. These will be discussed later as the opportunities arise.

Using this model, an alternative method for approximating both the radiation resistance and reactance of single-turn loop aerials up to antiresonance for specific geometries was published by Awadalla and Sharshar.[7] There, the radiation resistance is shown to be closely approximated as:

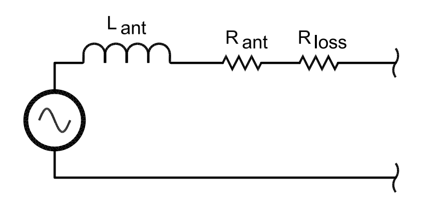

Fig 1—Loop aerial model (low frequency).

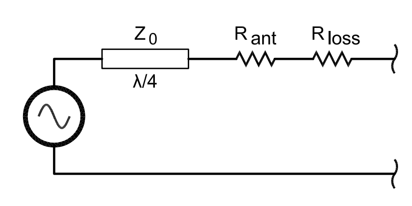

Fig 2—Loop aerial model (high frequency).

Table 1—Coefficients *a* and *b* for Equation 3

Configuration	*L/λ ≤ 0.2*		*0.2 ≤ L/λ ≤ 0.5*	
	a	*b*	*a*	*b*
Circular	1.793	3.928	1.722	3.676
Square (side driven)	1.126	3.950	1.073	3.271
Square (corner driven)	1.140	3.958	1.065	3.452
Triangular (side driven)	0.694	3.998	0.755	2.632
Triangular (corner driven)	0.688	3.995	0.667	3.280
Hexagonal	1.588	4.293	1.385	3.525

$$R_{ant} = a \tan^b \left(\frac{k_0 L}{2}\right) \quad \text{(Eq 5)}$$

The wave number k_0 is defined as:

$$k_0 = \omega \sqrt{\mu_0 \varepsilon_0} \quad \text{(Eq 6)}$$

where ω is the frequency in radians/second and ε_0 is the permitivity of free space (8.8542×10^{-12} F/m). The coefficients a and b are dependent on both the perimeter length and the geometry of the aerial, a list of values being shown in Table 1. Despite its convenience and favorable comparison to the moment-method (*NEC*) solutions, Eq 5 is only useful in approximating the radiation resistance for loops having a perimeter length of less than λ/2. It does not help in determining either the loss resistance or the aerial efficiency. Awadalla and Sharshar do, however, provide a useful approximation of the aerial reactance, wherein they approximate the characteristic impedance, Z_0, of the λ/4 transmission line in the model of Fig 2 as being:

$$X = j Z_0 \tan \left(\frac{k_0 L}{2}\right) \quad \text{(Eq 7)}$$

where Z_0 is the equivalent impedance of the transmission line, defined as:

$$Z_0 = 276 \log \left(\frac{4 A}{L r}\right) \quad \text{(Eq 8)}$$

where r is the radius of the wire or tubing. These equations are shown to compare favorably with method moment (*NEC*) simulations within the prescribed bounds (see Note 7) and are very useful in the design of single-loop aerials.

Resistances of Large Loop Aerials

The equations shown thus far are very useful in approximating the impedance of single-turn loop aerials and the efficiency of small loop aerials. They do not help in determining the resistances and the efficiency of a loop aerial that is large with respect to wavelength or has multiple turns, which is a much more complicated matter.

For this purpose, a more exact and comprehensive determination of the resistances and efficiencies for circular-loop aerials can be made by a series of equations developed by T. L. Flaig at the Electroscience Laboratory of Ohio State University.[8,9] From these reports, the radiation resistance for a circular multiturn loop can be determined as:

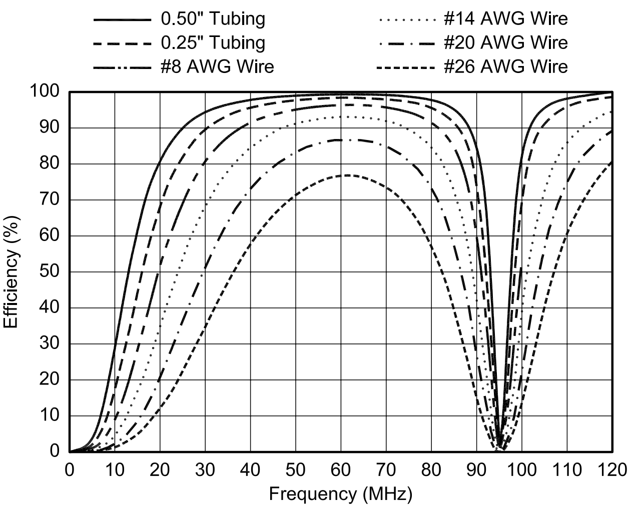

Fig 3—Aerial efficiency versus wire diameter.

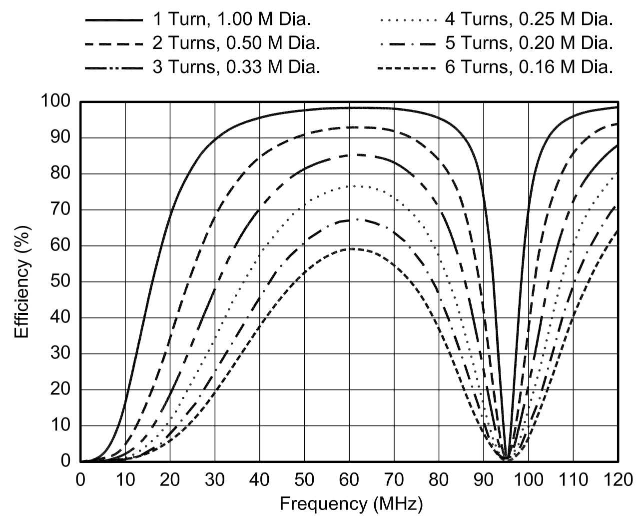

Fig 4—Aerial efficiency (constant wire length, 0.25-inch-diameter tubing).

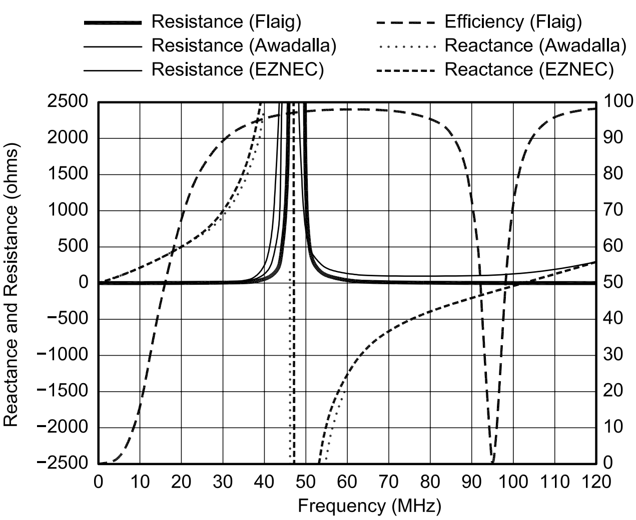

Fig 5—Aerial efficiency and impedances (analytical versus *EZNEC* free-space model).

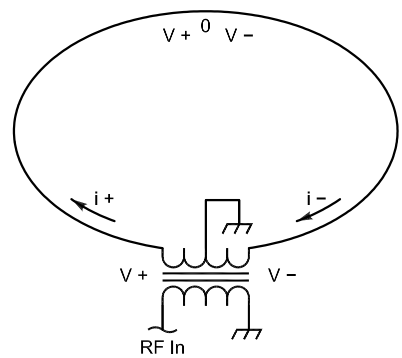

Fig 6—Loop aerial with balanced feed.

$$R_{\text{ant}} = \frac{8\pi^2 F^2 R^2}{9\times10^3}\tan^2(N\pi k_0 R) \qquad \text{(Eq 9)}$$

where F is the frequency in megahertz, R is the mean radius of the aerial and N is the number of turns in the aerial. Next, the loss resistance (for copper material) is determined by:

$$R_{\text{loss}} = \frac{4.16\times10^{-5}}{r}\,\frac{N\pi R\sqrt{F}}{\cos^2(N\pi k_0 R)}\left[1+\frac{\sin(2N\pi k_0 R)}{2N\pi k_0 R}\right] \qquad \text{(Eq 10)}$$

where r is the radius of the wire or tubing. By combining Eqs 4, 9 and 10, the efficiency of a circular loop aerial made with copper wire or tubing can be determined by:

$$Eff = \frac{\sin^2(N\pi k_0 R)}{\sin^2(N\pi k_0 R)+\dfrac{0.374N}{8\pi R r F^{1.5}}\left[1+\dfrac{\sin(2N\pi k_0 R)}{2N\pi k_0 R}\right]} \qquad \text{(Eq 11)}$$

More rigorous solutions for the resistances and reactance are available,[10, 11, 12] but are not as convenient as those presented by Flaig. As such, Eqs 9, 10 and 11 are well suited for analysis by way of spreadsheets such as *Excel*.

Analytical Evaluation

With suitable equations available, we can now embark on an analytical evaluation of the efficiencies and resistances of circular loop aerials. We begin the evaluation by first examining the effects that the size of the conductor has on the efficiency. Using Eq 11, the efficiency of a single-turn circular loop aerial having a diameter of 1 meter and a variety of conductor diameters ranging from #26 AWG to 1/4 inch (6 mm) was evaluated. The results of this initial evaluation are shown in Fig 3. Notice that the peak efficiency of this aerial decreases significantly as the wire diameter decreases. Also notice that the efficiency decreases rapidly below 20 MHz, becoming less than 5% for the 80-meter band. Now, consider what the efficiency of your 24-inch diameter commercial loop might be at 80 meters. (Think of a number less than 2%.)

We might want to consider making the aerial smaller

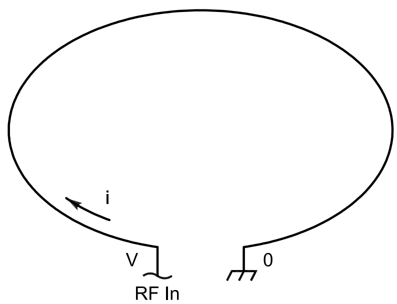

Fig 7—Loop aerial with unbalanced feed.

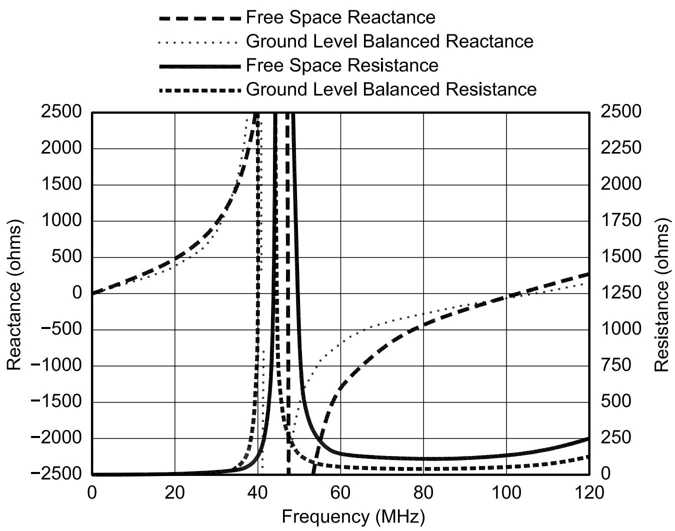

Fig 8—Free space versus ground environment impedances.

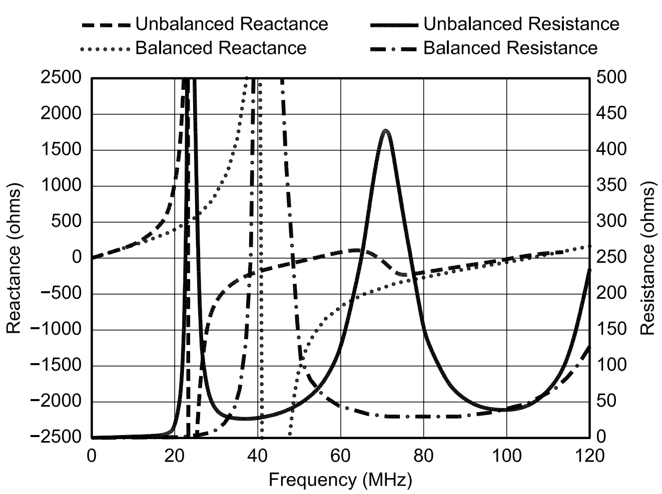

Fig 9—balanced versus unbalanced impedances.

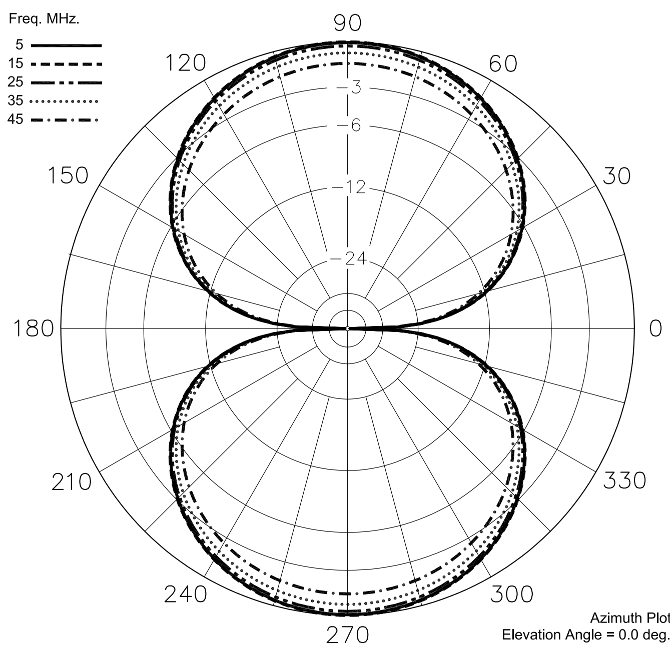

Fig 10—1.0-meter, 1-turn balanced aerial radiation patterns (*EZNEC*).

by increasing the number of turns while keeping the length of wire constant so as to retain the antiresonance and resonance characteristics. Might we make an efficient aerial of the same diameter while decreasing the antiresonance and resonance frequencies? Such an evaluation is shown in Fig 4, where increasing the number of turns actually degrades the efficiency in much the same manner as decreasing the wire diameter. This is not to say that multiturn loop aerials are impractical or that they should not be considered, but rather it is necessary to examine the consequences of making the aerial more compact by increasing the number of turns. The lower efficiencies of multiturn loop aerials can, in most circumstances be compensated to some degree by using larger diameter wire or tubing.

NEC Simulation

To gain some confidence in the analytical approaches described so far, it is useful for us to evaluate loop aerials with an *NEC* based simulation program, such as *EZNEC*. This will also allow us to evaluate aerials in proximity to ground, as the equations used so far are for free space and do not account for images or ground loading. At the same time, we need to recognize that *NEC* solutions will not give us any idea of the loss resistance, so we need to use the *NEC* modeling and the analytical solutions together to form a suitable basis for design.

To begin, a model for a single-turn loop aerial was formed and refined by comparison with the analytical equations and measured prototypes, the construction of which will be discussed later. This first model is a 1-meter diameter 12-sided polygon, similar to the first prototype when using wire, having a wire diameter of 6-mm (approximately 1/4-inch). The segmentation was adjusted at various frequency ranges to remain well within the guidelines that give good results.

The free-space *EZNEC* results for resistance compare very well with the radiation resistances obtained from Eqs 5 and 9 and are shown in Fig 5 along with the efficiencies derived by Eq 11 and the reactance derived by *EZNEC*. The resistances show that all three methods are in good agreement, the frequency of the antiresonance differing by less than 5% between them. From this and the comparisons that Flaig and Munk show for measured versus calculated efficiencies (see Notes 8 and 9), we can now have confidence in the radiation resistance and efficiency derived by their equations as well as the equations from Awadalla and Sharshar. We can do a more detailed analysis using *EZNEC* with good confidence that the efficiency will remain associated with the antiresonance frequency as the effects of ground proximity are brought into play.

Balanced versus Unbalanced Loop Aerials

The equations from both Flaig and Adawalla are meant for an aerial in free space, just as the model used so far in *EZNEC*. In free space, the aerial has no reference to ground and is thus balanced (assuming a single source is used in the *EZNEC* model). When the aerial is placed in a more realistic environment where a ground is present, we have the opportunity to examine the differences in performance between a balanced and an unbalanced aerial.

Let's begin by looking at the two configurations to better understand their differences. A balanced loop aerial is shown in Fig 6, in which the load (or source) is coupled to the aerial directly by way of a transformer having a center-tapped secondary winding, often referred to as a hybrid transformer or simply hybrid. With the secondary tap connected to ground, the aerial is now fed with two equal but opposite voltages and currents, the consequence of which is that the very top, or center of the aerial is a virtual ground. This is the property of the loop aerial that allows Awadalla and Sharshar to make their approximation by considering it to be a shorted transmission line having a length that is equal to one half the aerial perimeter (see Note 7).

An unbalanced loop aerial is shown in Fig 7. Here, the signal is coupled to the aerial at one end while the opposite end is grounded. The result of this configuration is that there is no longer a virtual null at the center, which negates the quarter-wave approximation of Awadalla and Sharshar. Instead, the loop aerial is now $\lambda/2$ long, which has

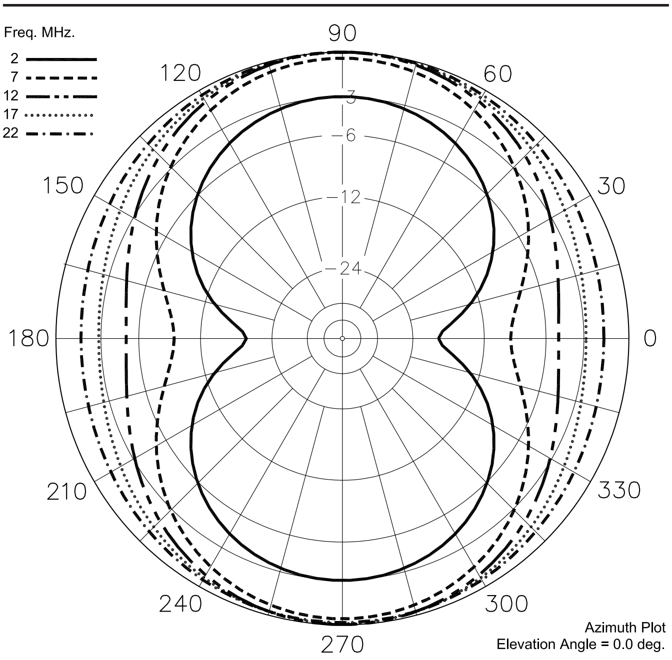

Fig 11—1.0-meter, 1-turn unbalanced aerial radiation patterns (*EZNEC*).

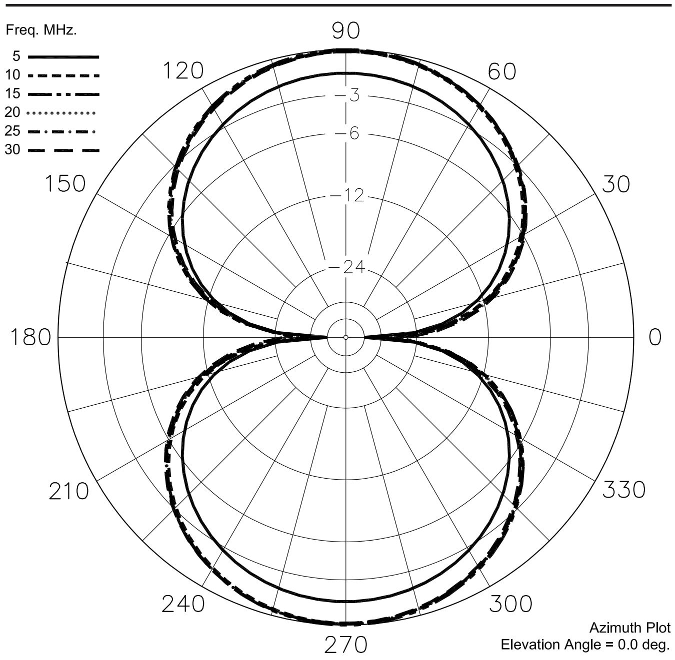

Fig 12—0.5-meter, 2-turn balanced aerial radiation patterns (*EZNEC*).

serious consequences on the impedance and the radiation pattern, as we are about to see.

By way of *EZNEC*, the loop aerial model used earlier is now placed in an environment having a perfect ground, with one end of the aerial connected to the ground, representing a worst-case application. This is not entirely unrealistic, as in the application of a compact aerial such as this we can easily anticipate that some users will mount the aerial in very close proximity to ground. For the balanced aerial, two sources of equal and opposite amplitude are used with the common point connected to ground. The result of this configuration is shown in Fig 8, where it is compared with the earlier free-space results, showing that the antiresonance frequency is shifted downward slightly as is the resonant frequency, as should be anticipated by virtue of parasitic loading to ground.

For the unbalanced aerial, one of the sources is removed, leaving the aerial with a single source on one side and the other side grounded, as described in Fig 7 earlier. In Fig 9, the results are compared with the balanced aerial results of Fig 8. It is clear that the antiresonant and resonant frequencies of the unbalanced aerial are now half of what they were for the balanced aerial, confirming that the unbalanced aerial is twice as long electrically as the balanced aerial. The low *Q* of the second antiresonance shows that the *Q* of the unbalanced aerial degrades with increasing frequency.

A comparison of the radiation patterns further reveals the degree to which unbalanced loop aerial operation degrades performance. The radiation patterns for the balanced loop aerial are shown in Fig 10, where the direction of the pattern is parallel with the plane of the aerial. Here we can see that the balanced loop aerial exhibits the deep, sharp nulls perpendicular to the plane of the aerial that are a characteristic of loops and make them popular for direction finding. The gain of the aerial degrades slightly as the frequency approaches antiresonance, but the quality of the nulls remains.

The radiation patterns for the unbalanced loop aerial are shown in Fig 11. The imbalance in the currents has led to a severe degradation in the quality and usefulness of the aerial, where the pattern nulls disappear rapidly at a low frequency and the aerial becomes virtually unidirectional as the frequency approaches antiresonance.

A subsequent model for a two-turn loop aerial was formed and used to evaluate the consequences of making a more compact aerial for the same range of frequencies. This second model is a 0.5-meter diameter 12-sided polygon having the same 6-mm wire diameter, the turns being spaced 30 mm apart, acknowledging the guidelines for minimizing the proximity effect given by Smith.[13, 14] The exact center of the aerial, at the bottom immediately adjacent to the feed points, is grounded to ensure exact balance. The radiation patterns are shown in Fig 12, and except for a slight reduction in the peak gain, this aerial remains suitable for consideration where circumstances dictate that a smaller aerial be used.

Construction Practices

Before we begin an experimental evaluation of circular loop aerials, notice that none of these equations nor any of the *NEC* programs (*EZNEC* included) account for the proximity of adjacent wires (in multiturn loop aerials) or the dielectric loading of the supporting structure, if any. We are all familiar with the effects that closely wound wires have on inductors, and the same is true here. The result of moving the wires closer together in a multiturn loop aerial is that the antiresonance frequency decreases but the resonance frequency remains virtually unaffected. We shall see this later, when we compare experimental results with those of *EZNEC*. Further, closely spaced conductors in multiturn loop aerials dramatically increase the loss resistance, and a spacing of at least five times the wire radius should be used to prevent this effect (see Notes 13 and 14). Hint: Do not use computer ribbon cable.

Since loop aerials have very high impedances at antiresonance, any dielectric loading from the supporting structure will effect the antiresonance frequency. Therefore, take care in the mechanical design, use insulating materials such as wood or plastic with low dielectric constants, and keep use of such materials to a minimum. Avoid metallic materials such as screws and nails. Rely instead on good woodworking joinery practices and proper adhesives. (Hint: Attaching copper tubing to a solid piece of plywood using EMT clamps with sheet metal screws is a bad idea.)

Larger diameter conductors, such as 1/4-inch (6-mm) or 1/2-inch (12-mm) copper tubing require less supporting material. Be mindful here that soft flexible copper tubing may contain lead, which greatly increases the bulk resistance. Large-diameter copper electrical wire and refrigeration tubing have good resistance characteristics and are therefore preferable materials. Rigid copper pipe is ideal if you can find some way of bending it into a uniform circle without collapsing it in the process.

Because of the low impedance of loop aerials, it is essential that all solder connections be done thoroughly and properly, using a good acid-free flux to ensure good wetting and preferably using a solder containing silver to avoid any lossy connections.

Experimental Results

With all of this analytical and mod-

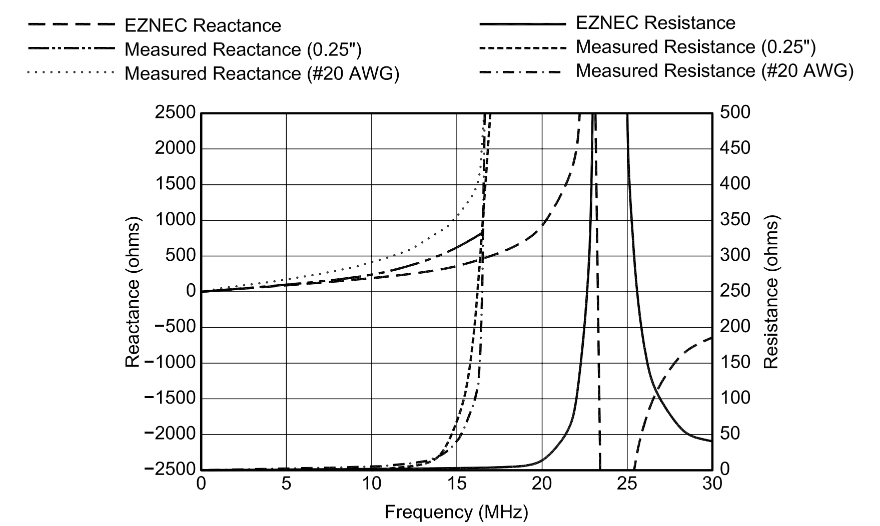

Fig 13—*EZNEC* and measured impedances for 1-meter, 1-turn unbalanced aerial at ground level.

eling information available, it is now safe to construct and test a prototype. For demonstration purposes, a single-turn aerial having a diameter of 1 meter was constructed using both #20 AWG wire and 1/4-inch refrigeration tubing. The supporting frame was made with a 24-inch-diameter circle of 3/4-inch-thick plywood to which arms made from 1/2-inch-thick plywood were attached at 30° intervals. The arms were glued to the disk and held in place with screws, which were later removed to reduce any parasitic loading.

Impedance measurements were made using a GR-1606A bridge and an HP-8407A vector network analyzer, and then comparing the two methods to correlate the results. This was necessary because of the low impedances at lower frequencies and the high impedances as the antiresonance is approached. Care was taken to ensure that the measurements were not affected by nearby objects. The presence of nearby AM broadcast stations made repeated measurements necessary to ensure accuracy.

Initial tests were made for the unbalanced aerial using #20 AWG wire for the conductor for later comparisons. The conductor was then replaced with more desirable 1/4-inch copper tubing. These test results are shown in Fig 13 along with the *EZNEC* simulation results for the unbalanced aerial model. Here we can see that the antiresonance frequency has been lowered by almost 25%, most of which can be attributed to the dielectric loading caused by the supporting frame. There is little difference in the antiresonance between the two conductors, and the sharp knee in the resistance is similar to that from the *EZNEC* simulation, showing that the aerial *Q* has not suffered appreciably from the loading.

In Fig 14, the *Q* of the aerial is calculated and compared between the two conductor materials, the 1/4-inch copper tubing having a *Q* almost twice that of the #20 AWG wire, especially at the lower frequencies. This verifies the earlier finding from Kraus' Eq 2 and Flaig's Eq 9 and 10 regarding the effect of the wire diameter on the resistances. This effect is well known in practice, and it is reassuring to know that an analytical method is available to aid in the design process.

Finally, the prototype aerial was configured for balanced feed using a three-winding hybrid transformer, the exact details of which will be covered in the design of the antenna amplifier/tuner in Part 2 of this series. The results in Fig 15 show that the antiresonance has been reduced by 32%, which is reasonably consistent with the reduction noticed for the unbalanced aerial test.

These tests confirm that an unbalanced loop aerial feed effectively reduces the resonance frequency to half that of the same aerial with a balanced feed, thereby verifying the *EZNEC* simulations used in the design process. To this degree, we can now have confidence in using the equations from the two analytical methods in conjunction with the simulation of *EZNEC* in the design of loop aerials.

Synopsis

Loop aerials are viable options when compact antennas are being considered. The results described here show that it is not unreasonable to construct such an aerial with design efficiencies in excess of 80% provided that certain design rules are given proper attention:

1. The diameter of a single-loop aerial should be approximately λ/20 to λ/10.
2. Use the fewest turns possible, preferably no more than two. If using two turns, ground the center to ensure good balance.
3. For multiturn loop aerials, separate the turns by no less than five times the conductor radius to minimize proximity effect.
4. Use the largest diameter conductor available, such as 1/4 or 1/2-inch copper tubing. Do not use tubing that is a copper/lead alloy.
5. Solder all junctions properly, preferably using silver solder to accommodate the low impedances.
6. Minimize support material to avoid parasitic loading.
7. Use a balanced feed.

With due attention to these details, it is easy for those having a good understanding of the mechanical aspects of aerial construction to make a loop aerial having good efficiency. In Part 2, we will cover the theory, design and operation of the antenna amplifier/tuner and the control unit.

Notes

[1]A. Boswell, "Loop Antennas in the 3-30 MHz Band," *Eighth International Conference on HF Radio Systems and Techniques*, 2000, pp 33-36.

[2]"Radio Instruments and Measurements," National Bureau of Standards Circular C74, 1937.

[3]A. J. Henk, "Loop Antennas: Fact, Not Fiction," *Radio Communication*, Sep 1991, pp 51-53 and Oct 1991, pp 47-50.

[4] J. D. Kraus, *Antennas*, 2nd ed. (New York: McGraw-Hill, 1988).

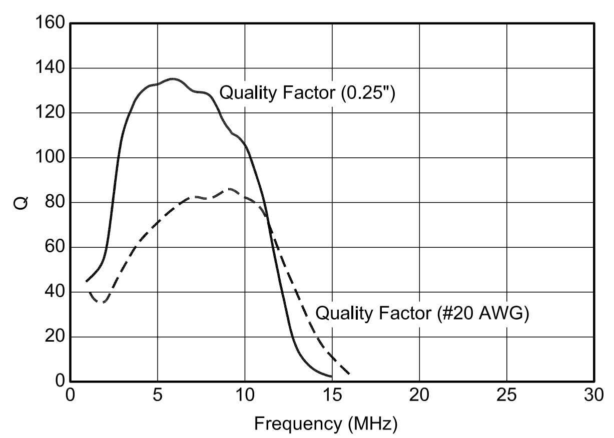

Fig 14—1-meter, 1-turn unbalanced antenna quality factor versus wire diameter.

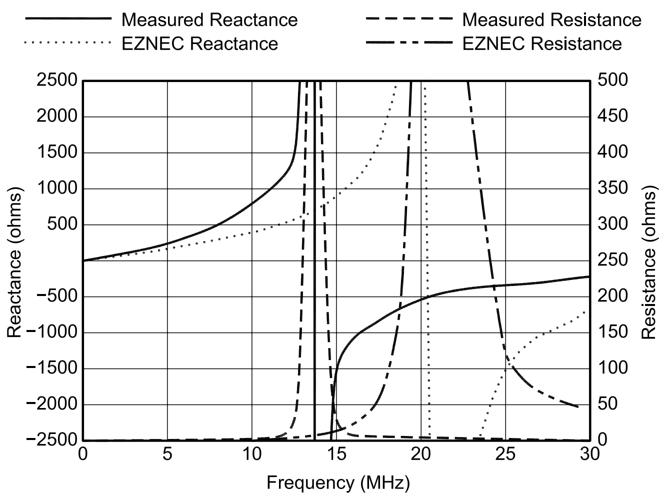

Fig 15—*EZNEC* and measured impedances for 1-meter, 1-turn balanced aerial at ground level.

[5]F. E. Terman, *Electronic and Radio Engineering*, 4th ed, (New York: McGraw-Hill, 1955).
[6]E. H. Newman, P. Bohley and C. H. Walter, "Two Methods for the Measurement of Antenna Efficiency," *IEEE Transactions on Antennas and Propagation*, Vol 23, No. 4, Jul 1975, pp 457-461.
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